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Optimal Reinsurance Arrangement for Fire Insurance

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ABSTRACT

The main purpose of this study was to set out a methodology to find out an optimal reinsurance arrangement from among Proportional and Non-Proportional reinsurance types for fire claims. The study starts by introducing reinsurance treaties which are widely used, Ruin probability in risk theory was used for the design of methodology. The adjustment coefficient, a useful quantity in risk theory, determines ruin probability of reinsurance type. The reinsurance type with the minimum ruin probability was selected as the optimal reinsurance type for fire claims. The calculation of adjustment coefficient requires the identification of an appropriate statistical distribution for fire claims. It is revealed from the literature that the four distributions, Weibull, Lognormal, Gamma and Exponential distributions are the candidate distributions for insurance claim data. Therefore the four distributions were fitted to fire claim data and the best statistical distribution was selected based on the Minimum Akaike's Information Criterion (A.I.C). The goodness of fit of the distribution was tested next graphically by using the Quantile-Quantile plots (Q-Q plots). Among the four distributions, the Gamma was identified as the most suitable distribution that fits fire insurance claim data at one of leading insurance companies in Sri Lanka. Moreover, Quota share (Proportional) reinsurance type was found to be optimal reinsurance arrangement for fire business class of the Company. For an insurance company, it is important to use reinsurance to transfer proportions of risk portfolios to other parties when the claims amounts are too large. However, it is company's responsibility to identify an appropriate reinsurance arrangement which optimizes profit. The results of this study help the company to eliminate the above burden since there were no previous studies which used a reliable methodology to find out an optimal reinsurance arrangement.

KEYWORDS: Reinsurance, Proportional, Non-Proportional

1 INTRODUCTION

"Reinsurance is the insurance of the risk borne by the insurer" (MAPFRE foundation, 2013).

In a reinsurance contract, one party (the reinsurer) for a certain premium agrees to indemnify another party (called the ceding company) for a specified part of its underwritten insurance risk. Hence reinsurance is a form of risk sharing mechanism between insurance companies.

For an insurance company, it is important to use reinsurance to transfer proportions of risk portfolios to other parties. It is arranged through some form of agreement in order to reduce the likelihood of having to pay a large obligation resulting from an insurance claim, which means that an insurance company can reduce the risks associated with underwritten policies by spreading risks across alternative institutions.

Fire insurance is used to cover damages to a property caused by fire. Fire insurance is a specialized form of insurance beyond property insurance. Generally a fire policy can insure a property against fire and lightning, riots and strikes, malicious damages, explosions, cyclone/ storm/ tempest/ flood etc.

Two types of reinsurance arrangements can be seen in the literature. They are the Proportional treaty and the Non - proportional treaty. Under the proportional treaty it has two categories, Quota Share and

Surplus and Excess-of-loss and Stop Loss are the categories of non-proportional treaties. A method has to be proposed to identify the optimal reinsurance type for the fire insurance business class from among Proportional and Non-proportional treaty. Therefore the objectives of this research are as follows:

1. To fit a statistical distribution for insurance claim severity in fire comprehensive policy and,
2. To find out the optimal reinsurance arrangement for the fire class from among Proportional and Non-proportional reinsurance types.

2 LITERATURE REVIEW

Achieng used actuarial modeling for insurance claim severity in motor comprehensive policy using industrial statistical distributions. In this study, actuarial modeling was used to fit a statistical distribution to motor claim data. The statistical distributions he tested were the Log-normal, Gamma, Exponential, and the Weibull. In the actuarial modeling process, a model family is selected first, from which, one distribution is to be selected as the best fitted distribution for the claim data. Then the parameters are estimated using the maximum likelihood estimation method, after which the log likelihood estimates are computed. Testing of the goodness of fit is then carried out using the A.I.C. and the Q-Q plots.

Fitdistrplus (An R package for fitting distributions-2014): the package fitdistrplus provides functions for fitting univariate distributions to different types of data (continuous censored or non-censored data and discrete data) and allows different estimation methods (maximum likelihood, moment matching, quantile matching and maximum goodness-of-fit estimation). Outputs of fitdist and fitdistcens functions are S3 objects, for which generic methods are provided, including summary, plot and quantile. This package also provides various functions to compare the fit of several distributions to the same data set and can handle bootstrap of parameter estimates. Detailed examples are given in food risk assessment, ecotoxicology and insurance contexts. This package is used for the advanced analysis.

Package "actuar" (Actuarial Functions and Heavy Tailed Distributions-2015): This package facilitates various actuarial science functionalities, mostly in the fields of loss distributions, risk theory (including ruin theory), simulation of compound hierarchical models and credibility theory. The package also features 17 probability laws commonly used in insurance, mostly heavy tailed distributions.

3 RESEARCH PROBLEM

The fire Insurance is a category of General Insurance which is used to cover damages to a property caused by fire. Therefore, large claims could happen in this fire business class. Because of that, insurance companies prefer to use a reinsurance arrangement to transfer proportions of risk portfolios to other parties. When using a reinsurance arrangement, it is very important to choose a profitable reinsurance arrangement (Optimal reinsurance arrangement) for the company.

Though the studies on Optimal Reinsurance arrangements for other categories of General Insurance are available in literature, only a few studies have been conducted on fire insurance. Therefore, in this study we investigated the optimal reinsurance arrangement for fire insurance among Proportional and Non-Proportional reinsurance types.

4 METHODOLOGY

Based on the company released data, the fire claims occurred during the business year 2012 at one of the leading insurance companies in Sri Lanka were used for this study. The candidate family of distributions for insurance data was selected by using the details found in the literature review. Normally, General insurance claim data follow special types of statistical

distributions. There are a number of parametric probability distributions as potential candidates for the generating mechanism of claim amounts. Mostly, data on general insurance are skewed to the right. Therefore, distributions that exhibit this characteristic can be used to model the fire claim severity. The Weibull, Lognormal, Gamma and Exponential Distributions were tested as candidate distribution for this claim data.

Due to heavy right tail in actual data, log transformation was applied to reduce skewness up to some extent and difficulties faced in distribution fitting. Out of the four probability distributions sampled, only one, which appears to fit the data set better than the rest, had to be chosen. Since the parameters were obtained using maximum likelihood, the criterion for choosing one distribution out of the four was also based on the values of the maximum likelihood estimates such that the larger the likelihood, the better the distribution fitted claim data.

It was assumed that none of the set of models considered was true. Therefore, selecting the best model is the main task. Just because a distribution possesses the highest log-likelihood out of the four distributions does not give sufficient evidence to determine whether it is the right distribution for the claim data. Therefore, an assessment was made to ascertain how best the distribution fitted the claim data by using the Akaike Information Criterion (A.I.C.) value. Then the classical goodness of fit was tested graphically.

Ruin probability in the risk theory was used to find out the optimal reinsurance arrangement. The reinsurance arrangement with the minimal ruin probability was taken as the optimal reinsurance scheme. For this study, quota share from proportional treaty and Excess-of-loss from non-proportional treaty were considered. Adjustment coefficients were calculated for each of these two treaties and compared in order to find the optimal reinsurance arrangement. The reinsurance arrangement with the maximum adjustment coefficient was the best, because the larger the adjustment coefficient, the smaller the ruin probability (Kolkovska, 2007).

5 RESULTS AND DISCUSSION

The actuarial modeling process started with the computation of summary statistics of the claim amounts. These are presented in Table 1 below.

Table1: Summary Statistics

N	1149
min	23.54
max	1.05e+08
median	29808
mean	430236.7
Estimated sd	3499738
Estimates skewness	24.24287
Estimated kurtosis	703.4151

This summary was necessary in pointing out the important features of the data. 1149 fire claims occurred in 2012 business year. The data were heavily skewed to right, which was an important feature in selecting the family of distributions to use.

The following four distributions were fitted to fire claim data separately and the summary of the estimates are shown in the following Table 2.

Table 2: Summary of the estimation results

Distributions	Log-likelihood	A.I.C	Parameters	
			Shape	scale
Weibull	-1461.629	2927.259	5.613624	4.89314
Lognormal	-1406.015	2816.03	0.0272627	0.183717
Gamma	-1391.42	2786.839	30.716906	6.748655
exponential	-2890.273	5782.546	0.2197053	-

Table 2 shows the computed log-likelihood values. From the tabulated statistics, Gamma distribution has the highest log-likelihood value of -1391.42 among the candidate distributions. Also it has the smallest A.I.C value. Hence, the Gamma distribution was the best fit of four distributions.

The following Figure 1 and 2 shows the classical goodness fit of the distributions.

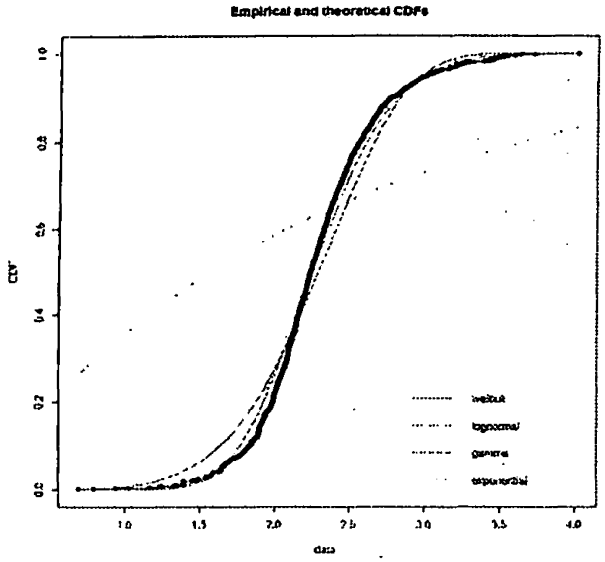


Figure 1: A plot of cumulative density functions against log transformed claim amounts

The graphical representation in Figure 1 shows the plot of C.D.F (Cumulative Density Function) of the four statistical candidate distributions. In comparison to the Empirical Distribution of Log transformed data, from the diagram, the Lognormal and Gamma distributions are relatively concluded to have best fits.

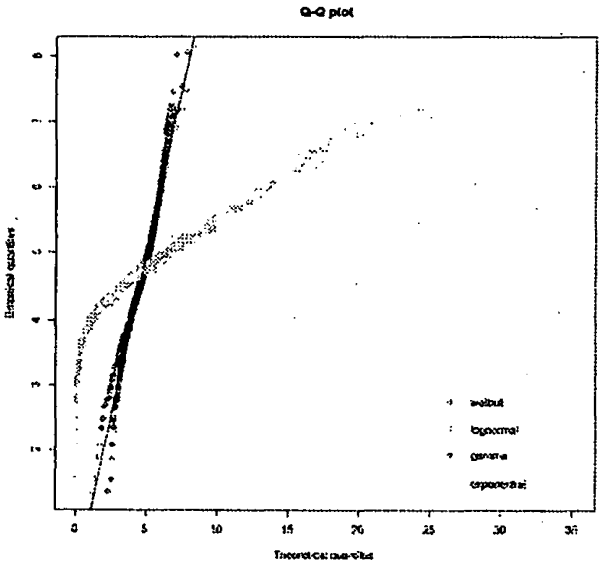
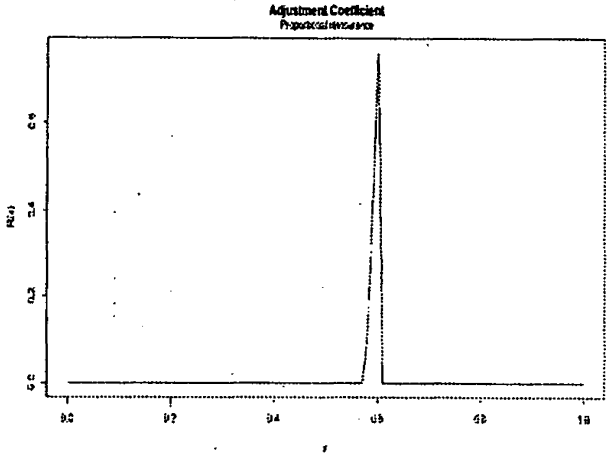


Figure 2: Quantile-Quantile Plot of candidate Distributions

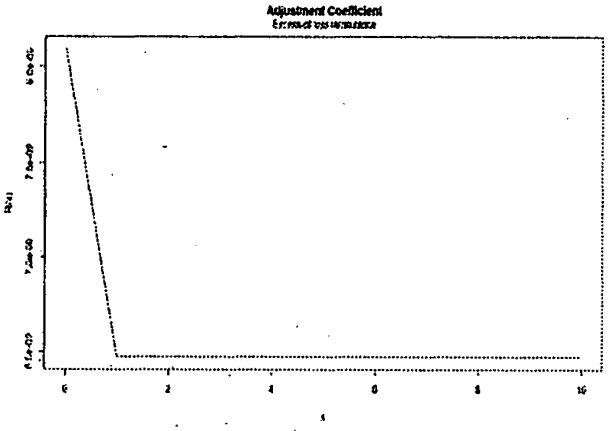
Figure 2 shows the Q-Q plots of candidate distributions. The Q-Q plot of the Exponential distribution is the worst fit because the majority of dots lie very far from the reference line.

Adjustment coefficients for the quota share and excess-of-loss reinsurance arrangement are illustrated in the following Figures 3 and 4 respectively. In both figures Y- axis represents the adjustment coefficient R(x).



X-(Quota share ratio)

Figure 3: Adjustment coefficient for the Quota Share (Proportional) Reinsurance Type



X-(Deductible Amount)

Figure 4: Adjustment coefficient for the Excess-of-loss (Non-Proportional) Reinsurance Type

As seen in Figures 3 and 4, the high adjustment coefficient value was presented in proportional reinsurance type at Quota Share ratio 0.6.

Identifying an optimal reinsurance arrangement using ruin theory was the main idea of this study. The data analysis of the study was heavily depended on the facilities available in the "adjcoef" function in the

"actuar" Package in R. Reinsurance arrangement with minimum ruin probability was the best. This study calculated adjustment coefficient to select the best reinsurance arrangement for the fire insurance since the maximum adjustment coefficient implied the minimum ruin probability. Only quota share and excess-of-loss treaties were compared for this study due to time limitation.

6 CONCLUSION

The main objective of this study was to identify the best reinsurance type which maximizes profit for the fire business class from among proportional and non-proportional treaties.

Among the four distributions Weibull, Lognormal, Gamma and Exponential, the Gamma was found to be the best probability distribution that fitted the fire insurance claim data.

From adjustment coefficients, obtained separately for the proportional and non-proportional reinsurance types, it is clear that, Quota share of proportional reinsurance was the most suitable type of reinsurance for this particular business class. Since there were no previously published studies which used reliable methodology to find an optimal reinsurance arrangement, the company can utilize the findings here in order to expect profitable business. Finally, for a future study they can try on an appropriateness of combination of quota share and surplus treaties as reinsurance arrangement for fire claims.

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